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X-Ray Spectrography by Transmission of a Non-Collimated Beam through a Curved Crystal

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Summary. - We describe a method, already described summarily (13), to obtain intense, narrow x-ray spectra using a curved crystal without collimation.

A wide beam of x-rays is incident on the convex face of a crystal, curved to follow a portion of a circular cylinder; the reflecting lattice planes will be more or less inclined to the face of the crystal. For an orientation of the crystal so that its lattice planes are parallel to the axis of the cylinder about which the crystal is curved, we obtain, on the concave side, some spectra which are extremely narrowly focused on a cylinder tangent internally to the curved crystal and with a radius 2 times smaller. Rays reflected at each Bragg angle will be focused at a point on this surface. The width of a spectral ray depends on the useful angular opening and the thickness of the crystal; it is given in terms of these parameters. The influence of defects in adjustment is discussed; it is always possible to realize the set-up by calculation without increasing the widths of the lines.

For a given source the width of the crystal determines the luminosity of the instrument. A comparison of the luminosity of a spectrograph using a curved crystal with one based on the method of Bragg, of the same radii and which would give the same spectral width for the same Bragg angle, shows that for a certain practical case, the time of exposure can be reduced from the order of an hour to the order of a minute.

The method described will allow, with a very simple technique, the recording of very intense spectra with a high separating power for hard x-rays (wavelengths shorter than about 1.5 Å). It can be applied to spectrographs using either photographic plates or ionization detectors. It can also be used to easily obtain narrowly monochromatic beams of x-rays. It is particularly suited to work with wide, weak sources such as secondary scattered radiation or fluorescence. It is very well suited to chemical analysis by x-ray emission or absorption.

The progress of technology has put at the disposal of specialists some x-ray spectrometers with which it is possible to determine characteristic wavelengths with a high precision.

The measurements made in some laboratories lead to sensitivities of a thousandth of an X-unit, i.e. 10^{-6} Å.(1) Besides these apparatus of high precision and increased separating power, it is very important for a large number of investigations to have access to apparatus giving a high intensity, but with relatively good image quality.

Any x-ray spectrometer using crystalline reflection contains, no matter what its operating principle, at least one narrow slit (or a knife edge attached to the crystal^(*)) and a crystal; the slit limits the width of the image and affects the luminosity which can only be further weakened by the use of several collimators in series; in order to increase the intensity it is necessary to widen the slit, which can't be done beyond a certain limit without degrading the readability of the images of the spectra. In the case of double crystal spectrometers(1) the additional reflection from the second crystal would result in an additional weakening of the beam if it did not allow the use of a wide enough slit without spoiling the resolution; this explains why these devices are relatively intense.

(*) Seeman's "Schneidenspektrograph".

Currently we know of only a very small number of spectrometers whose principle results in a *high intensity*. We cite the setup of *Soller*(2) where the radiation is channeled between a large number of long channels very parallel in order to increase the useful area of the crystal; a second collimator of the same sort gathers the reflected radiation and precedes the ionization chamber, which alone can be used for the measurements, photographic plates being excluded here because of the large width of the reflected beam; the setup is really a monochromator.

The spectrometer built by *Kirkpatrick and du Mond*(3) consists of a large number of elementary spectrometers (fifty) of reflection on a crystal of calcite with a knife edge on each crystal ("Seeman" type): the multiple reflected beams corresponding to a given Bragg angle are made to converge in a narrow common region, in order to contribute to the formation of a single spectral line; to do that we distribute the crystals regularly on the arc of a circle, which focuses all the reflected rays on a point of the same circle. The adjustment of such an apparatus is evidently a work requiring patience; the authors themselves were astonished at being able to do it in 2 months.(4)

Some attempts have been made to obtain spectra using curved crystals without any collimation.

The use of curved crystals goes back to the early days of x-ray spectroscopy. It was introduced to avoid the misorientation of the crystal. The idea came from *M. de Broglie* and was used in different setups.(3),(6),(7),(8),(9) A divergent beam *after passing through a slit* fell nearly tangentially on the *convex face* of a mica slab curved following a circular cylinder whose generators are parallel to the slit; the rays whose incidence on the *ordinary cleavage planes*, conforming to a portion of the cylinder, satisfy the Bragg condition for one of the wavelengths present in the beam are reflected; in fact this condition is satisfied only for a narrow zone along a given generator, and we obtain a spectrum of lines parallel to the slit, whose width depends on that of the slit and the distance from the plate to the crystal. In addition *R. Darbord*(10) had made an interesting

attempt to form the image of a slit by *reflection* of a diverging beam on the *concave side* of a mica slab.

These works do not profit from the essential interest of the use of curved crystalline slabs, that is the possibility of focusing on a given surface the different reflected beams corresponding to different Bragg angles and without narrow restriction of the incident beam. Some theoretical considerations and experimental trials were made in this direction.(11) *Johann*(12) was able to conceive and construct a spectrometer where the *reflection on the concave face* of a mica slab, curved following a portion of a circular cylinder, brings about for each Bragg angle a narrow focusing on a cylinder tangent internally to the first one and whose radius is two times smaller. No limiting of the incident beam by a thin slit is necessary any more for the formation of the spectra. All other things being equal, the spectral lines are narrower as the radiation is softer (larger Bragg angles), and the useful portion of the crystal is more restricted. It is impossible to use the spectrometer for the short wavelengths because of the divergence of the rays and the practical impossibility of attaining very small Bragg angles; these difficulties are besides exaggerated by the fact that the lattice constants of mica relative to the planes of ordinary cleavage which enter here are large (approximately 10 Å) and lead to very small Bragg angles; in addition this results in a weak dispersion.

In order to work in the domain of hard X-rays (1.5 Å and below, thus in a region including wavelengths often used in X-ray physics) I had the idea of trying a setup which is close to those described above but which differs significantly from them as we will see. The principle has been summarized in (13); it uses the *transmission* of a non-collimated beam across a crystal bent to the shape of part of a circular cylinder. *The radiation is incident on the convex surface of the crystal and the spectrum is formed on the concave side, the selective reflections taking place on the internal lattice planes oblique with respect to the surface of the cylinder.* Below I give a more detailed description.

Principle of the method. - Consider a plane parallel crystal limited, for example, by two principle cleavage planes and suppose that it can be bent to form part of a circular cylinder. If we do not exceed the limits of elasticity of the crystal we can assume that each small elementary crystalline cell remains non-deformed, and that only its global orientation changes; thus the angles of the lattice planes limited to this small grain with the plane normal to the faces of the crystal passing through the center of the grain are not modified. If the bending is done so that the intersections of the faces of the crystal with a system of lattice planes more or less oblique to these faces are parallel to the axis of the cylinder, after bending the normal to one of these planes at a given point will be located in a plane perpendicular to the axis of the cylinder.

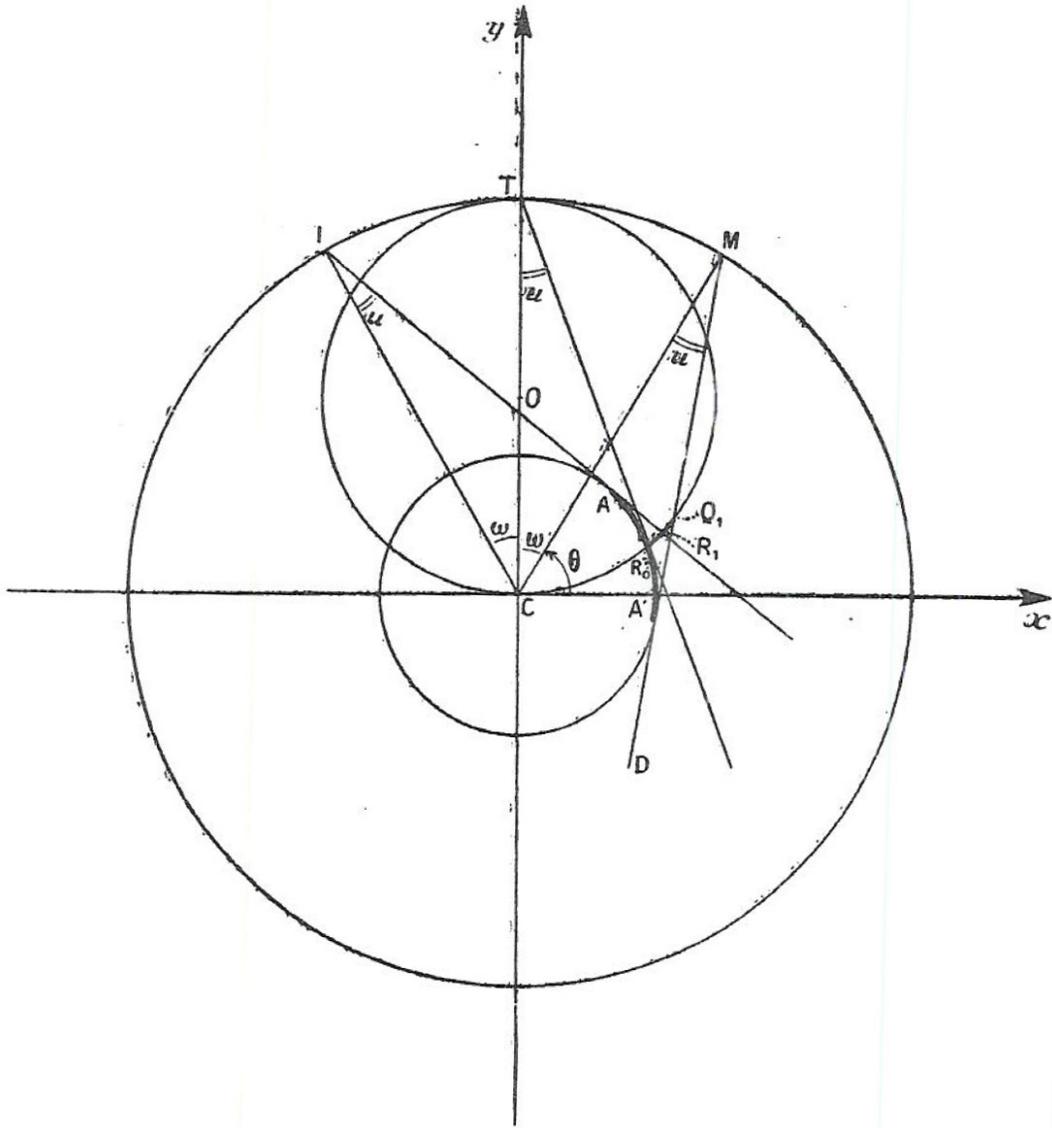


Fig. 1.

We assume it is so and that a wide beam of X-rays is incident on the convex face of the crystal; we consider a plane normal to the axis of the cylinder assuming, to begin, that the thickness of the crystal is negligible with respect to the radius of curvature. Refer to figure 1: the circle C represents the cross section of the cylinder on which the crystal is bent, the useful portion of which is the segment IM. At any point of incidence, M for example, the only rays that will be reflected are those which make an angle $90^\circ - \varphi$ with the normal to the reflecting plane (situated by hypothesis in the plane of the figure). The corresponding reflected ray MD also makes an angle $90^\circ - \varphi$ with that normal, thus an angle $\varphi + \alpha$ or $\varphi - \alpha$ with the radius CM, if α represents the angle of the considered lattice planes with the normals to the faces of the crystal. We see that all the reflected rays possible for a definite Bragg angle form a constant angle $u = |\varphi \pm \alpha|$ with the normals at the circle C whatever might be the incident directions present in the primary beam. The values of φ are given by the Bragg relation

$$n\lambda = 2d_n \sin\varphi_n$$

where we have to write the value of the lattice constant relative to the lattice planes of interest. The angle α follows from the crystal structure.

We are going to show that all the reflected rays corresponding to a given angle φ contribute to the formation of a narrow spectral line and that all the lines corresponding to different angles φ have as a locus a cylinder tangent to the crystal cylinder and having a diameter equal to the radius of the latter.

Let u be the angle of a reflected wave D with the radius of the circle C, inclined by θ with respect to the axis Cx (fig. 1); then the equation of a line D which passes through a point on the circle C (of radius R) with coordinates

$$\begin{aligned} x_0 &= R \cos\theta \\ y_0 &= R \sin\theta \end{aligned}$$

and which makes an angle equal to $u + \theta$ with Cx is

$$y - R \sin\theta - (x - R \cos\theta) \tan(\theta + u) = 0. \quad (1)$$

To find the envelope of the line D when θ varies, that is for a point of incidence which moves on the circle C, or in other words to find the caustic of the reflected rays, we associate with equation 1 the same equation differentiated with respect to θ

$$(x - R \cos\theta)/\cos^2(\theta + u) + R \tan(\theta + u) \sin\theta + R \cos\theta = 0 \quad (2)$$

and solve the system (1) (2) for x and y . We find

$$\begin{aligned} x &= R \sin u \sin(\theta + u) \\ y &= R \sin u \cos(\theta + u). \end{aligned}$$

This is the parametric representation of a circle with center C and radius $R \sin u$ (*). In practice the useful portion of the crystal is reduced to a slightly opened arc of which the middle is, for example, the point T on Cy for which $\theta=90^\circ$, and the lines D corresponding to an given angle u all will pass in the immediate neighborhood of the point

$$\begin{aligned}x &= R \sin u \cos u \\y &= -R \sin^2 u.\end{aligned}$$

When u varies, the locus of this point is a circle with equation

$$x^2 + (y - R/2)^2 = R^2/4$$

with center O at coordinates

$$\begin{aligned}x &= 0 \\y &= R/2\end{aligned}$$

thus situated in the center of CT, and having radius $R/2$, thus tangent to the circle C at T. There is a “focusing” of the spectra on this circle which we will call the “focusing circle”.

(*) In particular we find that the extensions of the lattice planes envelop a circle with center C and radius $R \sin \alpha$. The envelope of rays incident at a constant angle ν (If $u = |\varphi \pm \alpha|$, $\nu = |\varphi \mp \alpha|$) on some rays (radii) of the circle C is equally a circle with center C and radius $R \sin \nu$.

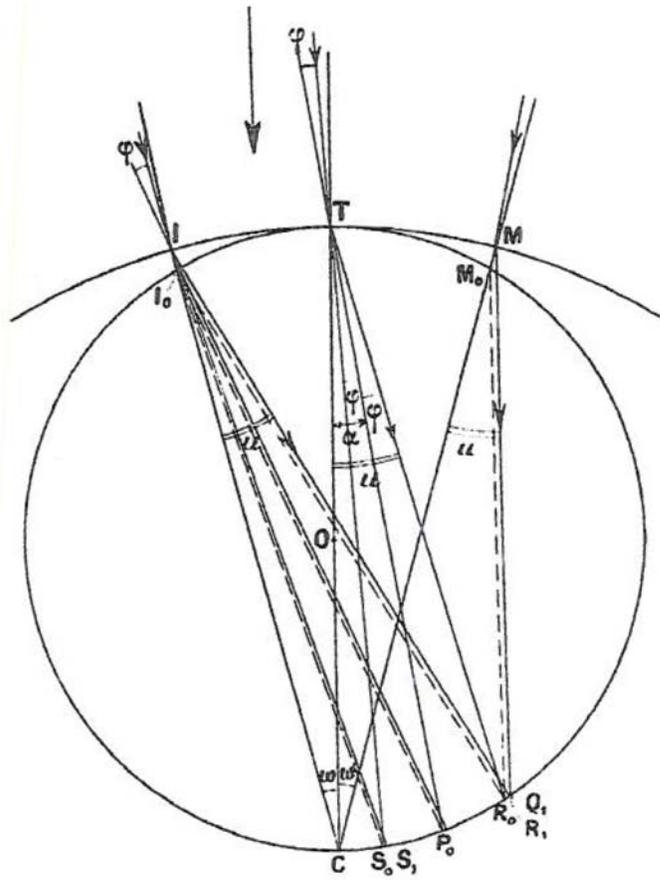


Fig. 2.
 $u = \varphi + \alpha$ $\varphi < \alpha$

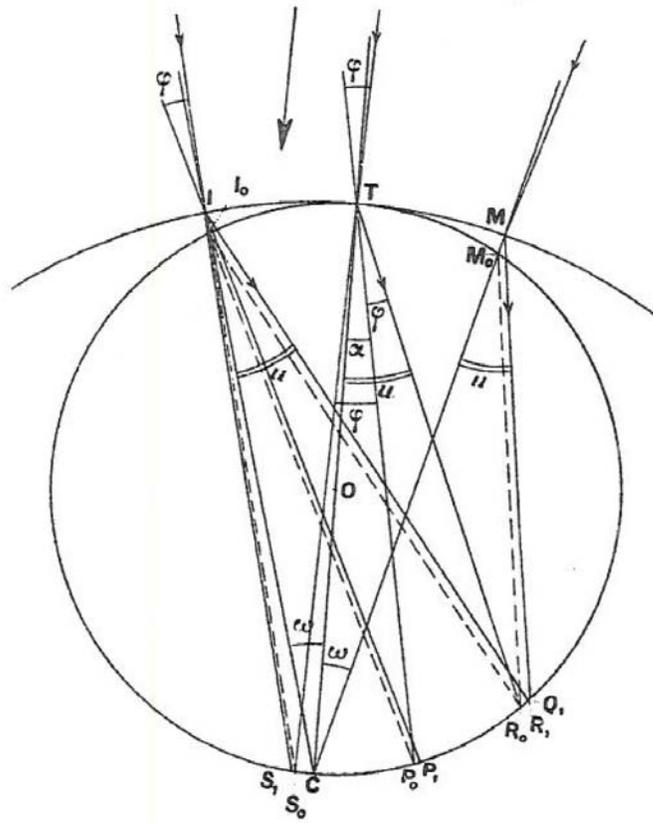


Fig. 3.

$$u = \varphi + \alpha \quad \varphi > \alpha$$

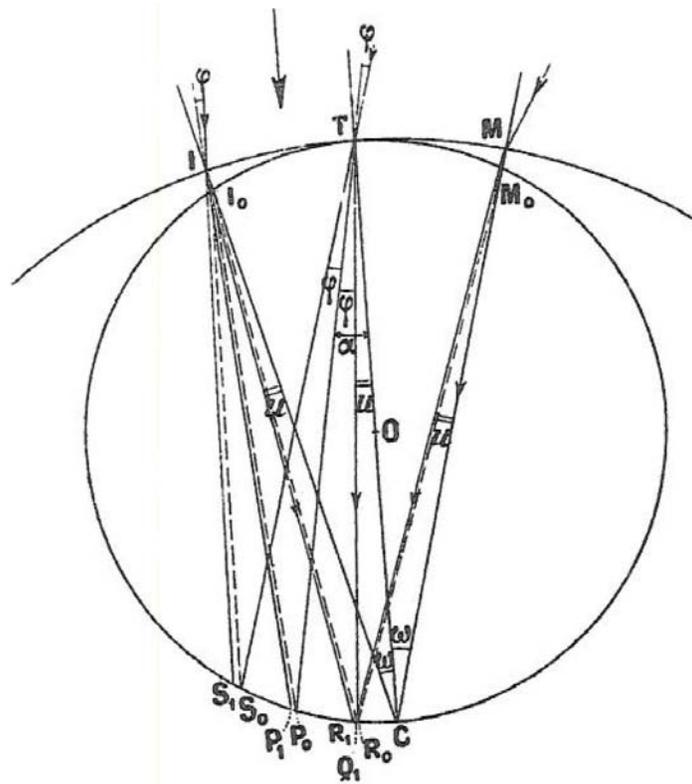


Fig. 4.

$$u = \varphi - \alpha \quad \varphi < \alpha$$

In figures 2, 3, 4 and 5 we show simple geometric constructions for the different cases which can be produced according to the conditions of incidence and the relative values of α and φ . The construction of the figures is aided by a given angle formed at a point such as I with the radius CI of the circle C, of the point I_o of intersection of CI with the circle O; thus the lines IS_1 (ray directly transmitted), IP_1 (trace of the reflecting plane), IR_1 (reflected ray), are obtained by drawing parallels to the lines I_oS_o , I_oP_o , I_oR_o , if TS_o , TI_o , TR_o , represent respectively the directions of the ray transmitted without deflection, the trace of the reflecting plane and the reflected ray relative to the point of tangency T.

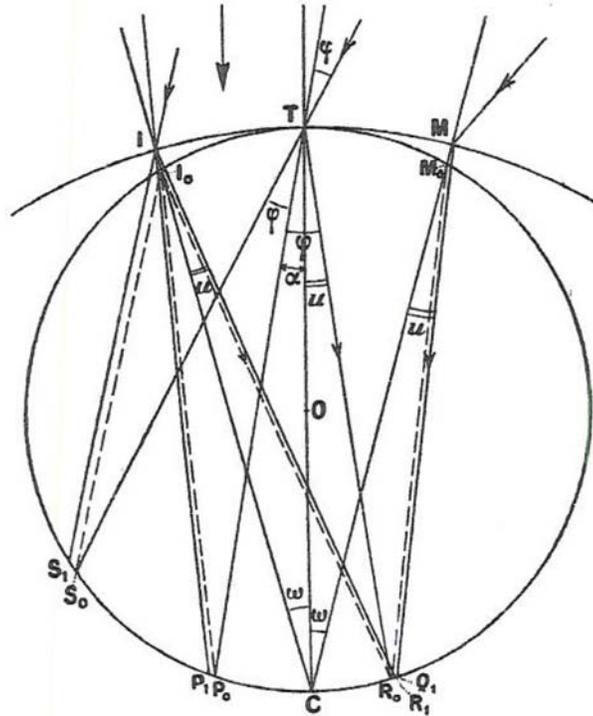


Fig. 5.

$$u = \varphi - \alpha \quad \varphi > \alpha$$

Similarly a given point of incidence M gives rise to a reflected ray MQ_1 parallel to M_oR_o .

All the rays reflected at a given angle φ converge along the arc $R_oR_1Q_1$ whose width is determined at first sight only by the useful opening of the crystal angle $ICM=2\omega$. If $u=0$, that is $\varphi=\alpha$ in the case where $u=(\varphi-\alpha)$, there is a focus at C.

These considerations have served as the basis of spectrographs where the crystal is curved along a portion of a cylinder such as CIM and a photographic film is rolled along the "focusing cylinder".

Latitude of adjustment. - While an exact placement of the film on the "focusing cylinder" will favor the narrowness of the lines, we have been able to practically obtain some very nice spectra with fine lines even with an approximate adjustment. The explanation lies in the fact that the concentration of the radiation takes place along the whole arc AA' (figure

1); any sensitive surface placed so as to cut this arc will show after development a narrow darkening with a clear border towards the interior of the circle C.

With the preceding notation the coordinates of A are

$$\begin{aligned}x &= R \sin u \cos(\omega+u) \\y &= R \sin u \sin(\omega+u)\end{aligned}$$

and of A'

$$\begin{aligned}x' &= R \sin u \cos(u-\omega) \\y' &= R \sin u \sin(u-\omega)\end{aligned}$$

and the distance along which it is possible to displace a photographic plate parallel to Cx is equal to

$$y - y' = 2 R \sin^2 u \sin \omega. \quad (I)$$

Dispersion on the focusing circle. - In all the cases in the figures, the clear boundaries (R_o) of two spectral rays corresponding to the Bragg angles φ_1 and φ_2 are seen from T under an angle $\varphi_2 - \varphi_1$, from O under an angle $2(\varphi_2 - \varphi_1)$ and their linear separation is equal to

$$\Psi = R (\varphi_2 - \varphi_1) \quad (II)$$

However $n\lambda = 2d_n \sin\varphi$

or $nd\lambda = 2d_n \cos\varphi d\varphi$

$$d\lambda/\lambda = \cot\varphi d\varphi$$

For two neighboring rays of wavelength λ_2 and λ_1 corresponding to angles φ_2 and φ_1 we can write

$$(\lambda_2 - \lambda_1)/\lambda_{\text{mean}} = \cot\varphi \Psi/R \quad (II')$$

This formula allows the calculation of the linear separation of two different wavelengths for example by 1 U.X. in the neighborhood of λ U.X., or inversely the dispersion in X units per mm near λ according to

$$\Psi = R (\Delta\lambda/\lambda) \tan\varphi \quad (II'')$$

Width of a spectral line on the focusing cylinder - case of an infinitely thin crystal. - We limit ourselves to an approximate calculation based on the figures 2, 3, 4 or 5, but the complete analytic calculation presents no difficulty.

Let ω be the useful angular semi-opening of the crystal $ICT=TCM=\omega$; if we designate by Δ the lengths of the small segments II_o , and MM_o , we easily calculate that

$$\Delta = 2R \sin^2(\omega/2).$$

The maximum spreading of a ray starting from the critical point R_o depends on the case IR_1 or MQ_1 ; it is given in general by

$$\ell = \Delta \sin u / \cos(\omega+u) = 2R \sin^2(\omega/2) \sin u / \cos(\omega+u) \quad (\text{III a})$$

and remains smaller than

$$\ell = 2R \sin^2(\omega/2) \tan(\omega+u). \quad (\text{III b})$$

If ω is very small, one can use the expression

$$\ell = R (\omega^2/2) \tan u. \quad (\text{III e})$$

If we designate by o the total useful linear opening of the crystal

$$o = 2 R \omega$$

we can write

$$\ell = (o^2/8R) \tan(\omega+u) \quad (\text{III'b})$$

or even

$$\ell = (o^2/8R) \tan u. \quad (\text{III'c})$$

A line is thus much more narrow as the crystal is less open and when it is located nearer to the center C ; we note that the case $u=\varphi+\alpha$ is disadvantaged and that it is good to avoid it in practice.

Influence of the crystal thickness (fig. 6). - The existence of a crystal thickness e towards the exterior of the circle C produces a unilateral widening of the spectral line on one side or the other of R_o , due to the points of incidence situated on the extreme radii CI and CM equal to

$$e \sin u / \cos(\omega+u).$$

The existence of a thickness e' towards the interior of the circle C produces a widening on one side or the other of the divide R_o (which is then absorbed in the total width of the line) with value

$$e' \tan u$$

corresponding to points of incidence situated on CT .

The influence of the total thickness $e+e'=E$ of the crystal (figure 6, 1) is a light broadening on the side of R_0 equal to $e' \tan u$, plus another broadening on the side of the initial spreading equal to $e \sin u / \cos(\omega+u)$.

The total width of a line is thus

$$l = e' \tan u + [e + 2R \sin^2(\omega/2)] \sin u / \cos(\omega+u) \quad (\text{IV a})$$

or, if ω is small enough

$$l = (e + e' + R \omega^2/2) \tan u \quad (\text{IV b})$$

or on substituting $\sigma=2R\omega$

$$l = (E + \sigma^2/8R) \tan u. \quad (\text{IV'})$$

For the same angular opening of the crystal, the broadening depends only on the thickness E and not on the radius of curvature, while the width due to the opening of an infinitely thin crystal is proportional to R ; depending on the case, one term or the other can dominate. We will see some examples. It is advantageous to give, if possible, the same order of magnitude to both terms. For a ray in the neighborhood of the center C , the influence of the width is negligible.

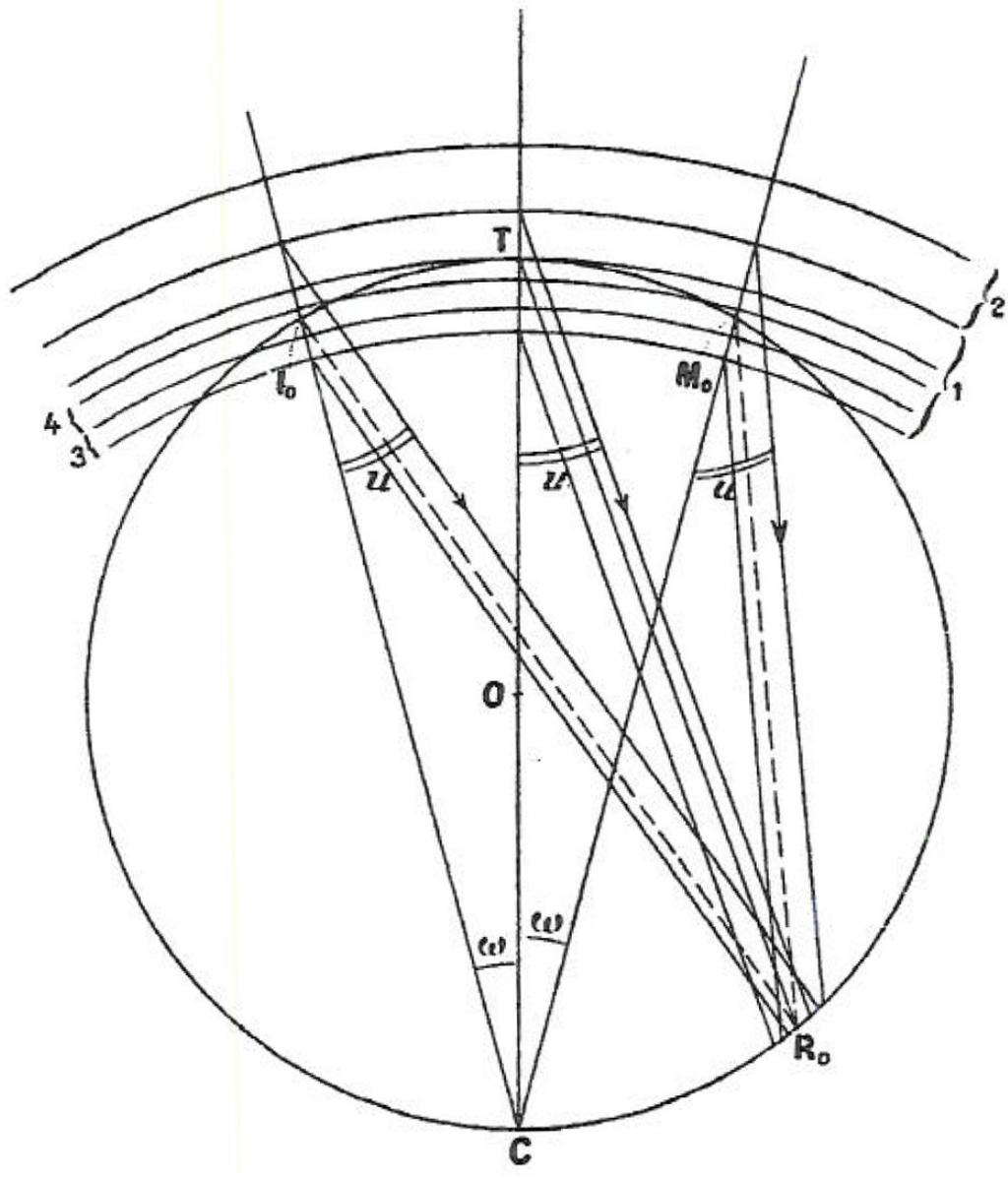


Fig. 6.

Influence of an error in adjustment. - The calculations made in the preceding paragraph can serve to determine the influence of an error in adjustment, that is of a *misplacement of the center of the cylinder carrying the film with respect to the focusing cylinder.*

The influence of the thickness of the film with double emulsion acts in the same sense. In the following summary we have taken the displacement relative to the crystal.

1. Displacement δ parallel to the internal face of the crystal towards the exterior of the circle C. Figure 6, 2. - R_o is displaced by $\delta \tan u$. The total width of a ray is

$$(\delta + E + \Delta) \sin u / \cos(\omega+u) - \delta \tan u. \quad (V)$$

2. Displacement δ' of the internal face of the crystal towards the interior of the circle C. - R_o is displaced by $\delta' \tan u$.

a) Figure 6, 1 : e and e' from different sides of T. R_o is immersed in the total width of the ray. The total width of a ray is

$$(E + \Delta) \sin u / \cos(\omega+u). \quad (VI)$$

b) Figure 6, 3 : $\delta' - E > \Delta$. R_o is immersed in the total width of the ray. The total width of a ray is

$$\delta' \tan u - [(\delta' - E) - \Delta] \sin u / \cos(\omega-u) \quad (VII)$$

c) Figure 6, 4 : $\delta' - E < \Delta$. The spectral ray keeps a clear border R_o in the direction of C. The total width of a ray is

$$\delta' \tan u + [\Delta - (\delta' - E)] \sin u / \cos(\omega+u). \quad (VIII)$$

We note that if δ and δ' remain small, and for the values of u generally used, these additional causes of broadening remain very small with respect to the width due to the thickness E and the opening of the crystal; it follows that it *is possible to realize the centering of the film carrier by construction.*

Influence of a defect of construction. - In a spectrograph based on this principle the curvature of the photographic film and the crystal are realized by construction (calculation). Let's suppose that there is a difference ε between the radius R of the crystal and the diameter $2r$ of the film, and refer to Figure 7.

a) $\varepsilon = R - 2r$ $R > 2r$. The film being applied on the circle O' results in each spectral ray being broadened on one side and the other of the point, R_o of intersection of CR_o with the circle O' . If we only consider regions of the film not too far from C' we can

assume that the broadening is equal to the length of the arc rq ; by noting that $\text{arc}(I_0R_0T) = \text{arc}(TR_0M_0) = \omega$ and by assimilating the arc to the tangent we find

$$r q = 2 R_0 R'_0 \tan \omega$$

However $R_0 R'_0 = T R_0 - T R'_0 = \varepsilon \cos u$.

The broadening is approximately

$$2 \varepsilon \tan \omega \cos u.$$

It is maximum for $u = 0$ that is for a ray which is focused at C; it is then symmetric around C' and has for a total value

$$2 \varepsilon \tan \omega,$$

where, if ω remains small

$$2 \varepsilon \omega = (o/R) \varepsilon. \tag{IX'}$$

b) $\varepsilon = 2r - R$ $R < 2r$. - The film is rolled on the circle O". The broadening for a ray focused on C is maximum and the upper limit of the broadening is

$$2 \varepsilon \omega = (o/R) \varepsilon.$$

This broadening remains equally small; practically we have taken $R = 200$ mm and $o = 20$ mm. However it is possible to obtain, without mechanical difficulty a precision of the order of a hundredth of a mm for the equality $R = 2r$. If $\varepsilon = 0.05$ mm, the corresponding broadening is approximately 0.005 mm and thus essentially negligible with respect to the broadening of the photographic darkening.

If $R = 100$ $o = 2$ $\varepsilon = 0.1$ mm

$$(o/R) \varepsilon = (0.1 \times 2) / 100 = 0.002 \text{ mm.}$$

Luminosity of the apparatus. - In a cross-section of the focusing cylinder the darkening of a sensitive layer along a spectral ray depends on the energy which falls in the neighborhood of R_o , thus on the energy of the beam of incident rays that converge towards S_o ; this depends on the brightness of the source, and, for the same brightness, on the useful width of the source; if we suppose that the source covers the entire opening of the angle IS_oM at a distance D from the crystal, the energy which falls on R_o will be proportional to the total linear opening o of the crystal and inversely proportional to the distance from the source to the sensitive surface, $D + R \cos u$; if we assume the crystal to be infinitely thin the width of the ray formed on R_o is then

$$\ell = (o^2/8R) \tan(u+\omega).$$

For a Bragg spectrograph of radius $R/2$ with a slit at T and whose crystal would rotate around the axis O , in order to obtain a ray of the same width ℓ for the same angle ϕ , the effective width of the crystal should be equal to $\ell/\sin\phi$, if we neglect the penetration of the radiation in the crystal; the energy which would contribute to the formation of that ray would be proportional to $\ell/\sin\phi$ and inversely proportional to $D + R$; if we neglect the differences of reflecting power of the two crystals, and if we note that the distances from the source to the sensitive layer are the same order of magnitude, we can say that the ratio of the respective energies which lead to the ray formation in the two apparatus, is

$$\rho = (o / \ell) \sin\phi$$

for the same exposure time. On substituting for ℓ the value

$$\ell = (o^2/8R) \tan(\alpha-\phi+\omega)$$

we find for ρ

$$\rho = (8R/o)\sin\phi/\tan(\alpha-\phi+\omega) = (4/\omega)\sin\phi/\tan(\alpha-\phi+\omega) \approx (4\phi/\omega)/(\alpha-\phi) \quad (X)$$

For $R = 20 \text{ cm}$ $\alpha = 10^\circ$ $\omega = 1^\circ$ $\phi = 6^\circ$, we find $\rho \approx 80$.

An exposure time on the order of an hour with Bragg's method would be less than a minute here.

These considerations have been made by assuming implicitly that the distances in the two apparatus are equal and the reflecting powers comparable.

Practical applications. - The practical application of the method requires the choice of a crystal easy to bend. The crystal slab is then bent in the inside of a sort of circular-cylindrical frame; the film is held the same way in a cylindrical frame; in many cases we can replace the film by a plate tangent to the focusing cylinder in the neighborhood of the region to study. The equality $R = 2r$ is obtained by construction.

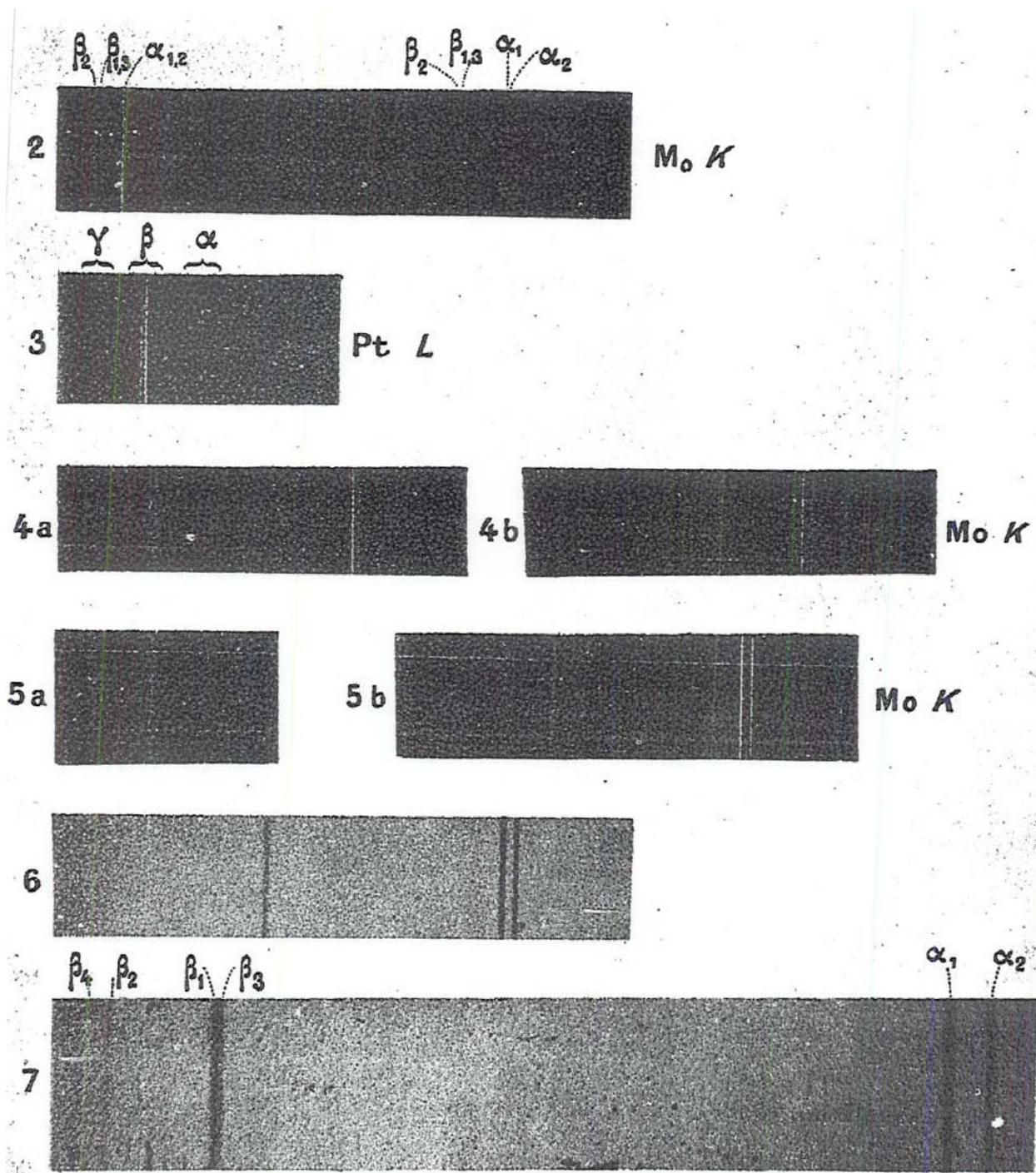
It is then necessary to make the following adjustments:

1. Locate the crystal in its frame in such a way that the reflecting lattice planes will be parallel to the generators of the cylinder.
2. Install the film on the focusing cylinder. This can be done by construction.
3. Install the rigid assembly of the crystal and the film and the lead shielding with respect to the source of radiation. This type of setup favors the use of a wide source, but we should naturally put two or three wide diaphragms in front of the crystal, and place the source so that the incident beam will include the maximum number of rays whose incidence corresponds to the Bragg angles desired while avoiding that the undeviated transmitted beam might fall on the region of the film where the spectra will be recorded. It is easy to determine the most favorable position of the source and the screens with the help of simple geometric considerations which we will not discuss here.

Use of mica. - Mica has given us some very good results. The unit cell is an inclined rhombic prism in the ordinary cleavage planes; the cell is a parallelogram of side a , diagonals a and $b = a\sqrt{3}$, with angles of 60 and 120 degree; the passage from one cell to the next is made by a translation c inclined by approximately 100 degrees to a , normal to b (¹¹). The values of the constants vary from on sample to another; the values of a oscillate around 5.2 Å, those of b around 9 Å, while the lattice distances corresponding to the planes of ordinary cleavage (corresponding to c) are approximately 9.9 Å. In our setup the lattice distance corresponding to a are important thus on the order of 5 Å, which gives a great advantage with respect to the setups that use the reflections on the ordinary planes of cleavage for the hard radiations.

The thickness of the slabs used has varied from 0.1 to 0.3 mm; they should be very flat before bending. To orient a slab in its support one can refer to the optic axes determined previously.

Plate I



Numerical applications. - We recall some formulas from above latitude of a point:

$$y - y' = 2R \sin^2 u \sin \omega. \quad (\text{I})$$

Dispersion in the neighborhood of a wavelength λ

$$\Psi = R (\Delta\lambda/\lambda) \tan \varphi. \quad (\text{II}'')$$

Width of a ray for a crystal thickness E

$$l = (E + \sigma^2/8R) \tan u. \quad (\text{IV}')$$

Remember that

$$\omega = \sigma^2/2R \quad u = | \varphi \pm \alpha |.$$

The equations II'' and IV' allow the calculation of one of the two values σ or E if you are given the other one of them as well as Ψ and l for some angles φ and u known.

Example. - If we put $\Psi = 10$ XU/mm – and $l = 0.05$ mm close to $\varphi = 8^\circ$ (for example for Mo K in the second order of the mica -) $\lambda = 700$ XU.

$$1 = R (10/700) (0.141) \quad (\text{II}'')$$

$$0.05 = (E + \sigma^2/8R) (0.035) \quad (\text{IV})$$

(II'') gives

$$R = 700/1.4 = 500 \text{ mm}.$$

If we put

$$E = 0.1 \text{ mm}.$$

(IV) gives

$$0.05 - 0.035 = \sigma^2/8R$$

$$\sigma^2 = 4000 \times 0.015 = 60.$$

$$\sigma = 7 \text{ mm}$$

Inversely, the application of these equations to the cases where the conditions were the following:

$$R = 20 \text{ cm} \quad o = 2 \text{ cm} \quad E = 0.125 \text{ mm.}$$

and $R = 100 \text{ cm} \quad o = 2 \text{ cm} \quad E = 0.2 \text{ mm.}$

has led to the numerical values in the following table, for $y - y'$, Ψ in mm per XU and l , for some spectral regions near to the K radiation of Cu, Br, Mb and W, in the second order (particularly intense). We have taken $\alpha = 10^\circ$ and $d = 5.25 \text{ \AA}$.

To determine l with $o = 2 \text{ cm}$ $R = 20 \text{ cm}$ the term $o^2/8R$ is not negligible; to the contrary, the thickness E counts alone for $R = 100 \text{ cm}$ and the same linear opening.

TABLE

Spectral regions around the 2nd order of

		RÉGIONS SPECTRALES ENVIRON 2 ^e ORDRE DE			
		Cu K	Br K	Mo K	W K
λ environ.		1 500	900	700	200 U.X
φ		16°	10°	8°	2° 30'
$u = \varphi \pm \alpha $	} ω	0,038	0,039	0,039	0,044 mm/U.X
$R = 20 \text{ cm}$					
$u = \varphi \pm \alpha $	} ω	0,490	0,195	0,195	0,220 mm/U.X
$R = 100 \text{ cm}$					
$u = \varphi + \alpha$	} $y - y'$	3,8	2,3	1,8	0,9 mm
$R = 20$					
et $R = 100$					
$R = 20 \text{ cm}$	} l	0,027	0,0212	0,019	0,013 mm
$E = 0,125 \text{ mm}$					
$u = \varphi - \alpha $	} $y - y'$	1	0	0,02	0,3 mm
$R = 20$					
et $R = 100$					
$R = 20 \text{ cm}$	} l	0,023	0	0,013	0,028 mm
$E = 0,125 \text{ mm}$					
$R = 100 \text{ cm}$	} l	0,024	0	0,007	0,026 mm
$E = 0,2 \text{ mm}$					

Use of other crystals - Slabs of mica have shown themselves easy to use and have given us some very good results. It can however be useful in certain cases to use some other crystals with the end of having some other lattice constants ($2d$ and α). The number of crystals able to support some moderate bending, on the order of one meter radius for example, is less restrained than you might believe.

Gypsum can be used with success under these bendings up to a thickness of approximately 1 mm. The structure of it has been studied with X rays by several authors (15); in order to orient it in its support it is sufficient to refer to the directions of secondary cleavage easy to recognize on the breaks. Note that the use of gypsum could be particularly advantageous in Johann's method, that is if we use the reflections on the planes of principle cleavage for which $d = 7.58 \text{ \AA}$, thus smaller than those of mica (approximately 10 \AA), especially for moderately hard radiation.

Description of photographs - We have reproduced on Plate 1 different spectra obtained by the method described, with the help of simple setups where in particular the position of the slab with respect to the focusing cylinder was imprecisely set.

All the spectra have been photographed on "Lumiere Opta" plates without any reinforcing screen.

Given the brightness of the apparatus the adjustment has been made with direct radiation using a fluorescent screen on which the spectra are strong even for some weak lines.

All the photographs have been over-exposed for reproduction; in direct radiation the spectra are formed with the desirable sharpness and finesse desirable for comparative measurements with energies on the order of 5 mA sec

1. Two orders of the K spectrum of rhodium excited with 40 kev, 5 mA obtained with a mica leaf 0.2 mm thick and radius of bending of 20 cm. Exposure time: 2 minutes.

2. Two orders of the K spectrum of molybdenum under the same experimental conditions.

3. Spectrum of the L fluorescence of platinum excited by the radiation of an anode of Mb excited by 40 kev, 10 mA; the distance from the anode to the radiator was about 25 cm, the distance from the radiator to the film was 35 cm. In these conditions the principle rays appeared after an exposure of 2 hours (using the same crystal as for 1 and 2). We note the weak relative intensity of the α rays relative to the β rays, due to the absorption of the long wavelengths in the thickness of the crystal.

4a and 4b. Two orders of the K spectrum of molybdenum obtained with a crystal of gypsum of thickness 0.5 mm curved on a radius of 1 meter. Exposure time: 4 min.

5a and 5b. Two orders of the K spectrum of molybdenum obtained with a mica crystal bent on a radius of 1 meter. In the higher order the separation $\beta_1 \beta_3$ is clearly visible to the naked eye. (We know that the wavelengths of these rays are respectively: $\lambda\beta_1 = 630.978$ and $\lambda\beta_2 = 631.543 \text{ XU}$) The dispersion in this order is approx 2.4 XU/mm.

6. The photograph 5a has been enlarged 4 times.

7. The photograph 5b has been enlarged 4 times.

Conclusion. - We have explained the principle of a simple method for obtaining some narrow X ray spectra with a resolving power which can become very high without

impacting the intensity. This method can be applied to the construction of spectrometers using films or photographic plates; an ionization chamber or a Geiger counter whose slit could be located on the focusing cylinder would allow ion counting measurements; in both cases the method allows a great luminosity and facilitates especially the study of X radiation coming from extended but weak sources, thus secondary radiations (scattering and fluorescence); the study of absorption spectra can be made photographically with very short exposure times, the secondary structures are seen to be perfectly clear and measurable. Any apparatus constructed on this principle is very well suited to chemical analysis by X rays (emission and absorption). Finally it is clear that the use of a thin slit placed on the focusing cylinder would transform the apparatus into a powerful monochromator.

This work has been carried out at the Laboratory of Chemical Physics since December 1931.

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References

- (1) We refer the reader to the following works. – A. Lindh, “*Röntgenspektroskopie*” *Hdb. d. Exp. Phys.* **24.2** (1930); M. Siegbahn “*Spektroskopie der Röntgenstrahlen*” 2nd Auflage Julius Springer, 1931.
- (2) W. Soller, *Phys. Rev.* **24** (1924), p. 158.
- (3) J. DuMond and H. Kirkpatrick, *Rev. of Scient. Instr.* (N. S.), **1** (1930), p. 88.
- (4) J. DuMond and H. Kirkpatrick, *Phys. Rev.* **37** (1931), p. 136.
- (5) DeBroglie and Lindemann, *C. R.* **158** (1914), p. 944.
- (6) Rohmann, *Phys. Zeits.* **15** (1914), p. 510.
- (7) Gorton, *Phys. Rev.* **7** (1916), p. 334.
- (8) Siegbahn, *Phys. Rev.* **8** (1916), p. 320.
- (9) P. Cerman, *Phys. Zeits.* **17** (1916). pp. 405 and 556.
- (10) R. Darbord, *J. Phys. and Rad.* **3** (1922), p. 218.
- (11) See on this subject. – Gouy, *Ann. de Phys.* (9), **5** (1916), p. 241; E. Dershen, C. T. Dozier, *Phys. Rev.* **17** (1921), p. 519; and the article of W. Linnik, W. Lashkarew, *Ukrain. Phys. Abh.* **1** (1926), p. 5, which it was not possible for me to consult.
- (12) H.-H. Johann, *Z. f. Phys.* **69** (1931), pp. 185-206.
- (13) Y. Cauchois, *C. R.*, t. **194** (1932), p. 362; t. **194** (1932), p. 1479.
- (14) Ch. Mauguin, *C. R.* **185** (1927), p. 288; **186** (1928), pp. 879 and 1134.
- (15) P.-P. Ewald, C. Hermann, *Strukturbericht (Akad-Verlagsges-Leipzig, 1931)*, p. 388.

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